Stochastic Optimization of Electric Vehicle Charging Stations

Georgios Chrysanidis, Dimitrios Kosmanos, Antonios Argyriou and Leandros Maglaras

Abstract—With range anxiety becoming the everyday problem for Battery Electric Vehicles (BEVs) owners, even more research is being conducted in the field of BEV charging and Charging Stations (CSs) scheduling optimization. In this context, our work addresses the problem of BEV charging in an urban environment with no a-priori knowledge of vehicle arrivals. Our system is modeled as a $M/G/K$ queuing system. Two adaptive charging algorithms are proposed, both of them relying on queue stability. The first one charges BEVs up to a percentage of their maximum capacity when charging queues become unstable. The second one when detects instability charges BEVs sufficiently enough to reach their next destination. Both algorithms can be used in combination with an admission control algorithm that does not allow BEVs that do not fulfill certain criteria into the charging stations. The First-Come-First-Serve (FCFS) algorithm is directly compared to our proposed algorithms, with prominent improvement concerning congestion in charging stations and waiting time of electric vehicles.

I. INTRODUCTION

As of 2017 a total of 3 million Electric Vehicles (EVs) have been sold worldwide with an increase of 50% in the sales just between 2016 and 2017. The predictions, based on the legislation voted and the constraints imposed by the European Union and other developed countries, are that by 2030 more than 130 million EVs will have made their way in the market [1]. Electric cars have the potential to reduce carbon emissions, local air pollution and the reliance on imported oil [2]. The turn of the automotive industry in the all-electric car is unprecedented [3] and as a result, a vast amount of resources is being invested in the development of Battery Electric Vehicles (BEVs). The market is still growing and there are many opportunities for innovation and profit. The impact of this turn is obvious in terms of Electric Vehicle Supply Equipment (EVSE) increased availability and rapid battery development. The fact that all major automotive companies have set the goal for electrification of vehicles can also be seen by the fact that the development of Internal Combustion Engines (ICE) has dramatically slowed down, with some companies soon retiring them completely. This shift will drastically change the driving habits of millions of people as both the range and charging time of EVs are still not comparable to those of an ICE vehicle [4].

To address this issue, in this paper we attack the problem of EV charging in urban environments, by reducing the time an EV is waiting to be charged. We assume a stochastic modeling of vehicle arrivals to our charging system. We adopt an adaptive queuing-based approach by scheduling in a way that keeps all the charging queues stable [5]. We propose two algorithms for this purpose. The first algorithm adjusts the target charging percentage of each EV when the queue grows more than the service rate of the charging station. However, when the queue backlog is stable, each EV battery is charged at its maximum capacity. The second algorithm considers the distance that EVs need to cover for their next trip. Every time we observe queue growth, the system enters what we call a Next-Trip-Mode where each EV is charged just enough to reach its next destination. As in the first algorithm, when the queue backlogs are normal charging takes place at the maximum EV battery capacity. The main advantage of our approach is that we use an adaptive technique to minimize the waiting time for charging the vehicles. Second, we do not need extra infrastructure costs for the installation of multiple Charging Stations (CSs). Third, we model and evaluate the system based on realistic assumptions collected from the most recent trends reported in the EV industry.

The rest of this paper is organized as follows. In Section II we provide a thorough analysis of the relevant bibliography on the field of EV charging. In Chapter III we describe the fundamental principles on which our model is built. In Chapter IV we formulate our optimization problem and set the constraints needed with a detailed explanation of our proposed algorithms, while the results of computer simulations are presented in Chapter V. Finally, Chapter VI concludes this work.

II. RELATED WORK

This rise in interest of both the society and the automotive industry has led into considerable research in the field of BEV charging and its integration into the existing infrastructure, as can be seen in [6] and [7]. Those detailed articles review the literature in BEV Charging Scheduling Optimization and present the problem formulation adopted in every case.

In [8] the authors focus on optimizing the driving range of EVs by deploying several mobile CSs beyond the static CSs. They formulate an optimization problem and due to its complexity they solve deterministic formulation of it that leads to significant extension of the driving range. In [9] the authors propose two techniques that exploit BEV charging during their workplace parking and utilize it through both Vehicle-to-Grid (V2G) and Grid-to-Vehicle (G2V) technologies. Their two strategies minimize daily cost and Peak-to-Average...
Battery Capacity (kWh) EE (kWh/km)

<table>
<thead>
<tr>
<th>EV Type</th>
<th>Capacity</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>20</td>
<td>0.1897</td>
</tr>
<tr>
<td>Mid-Size</td>
<td>50</td>
<td>0.1757</td>
</tr>
<tr>
<td>Large</td>
<td>75</td>
<td>0.2008</td>
</tr>
<tr>
<td>SUV</td>
<td>100</td>
<td>0.2487</td>
</tr>
</tbody>
</table>

### III. System Model

Our system consists of $N$ CSs co-located in an urban environment, each having $K$ chargers. We adopt a queuing model for characterizing the behavior of a CS. The system is modeled as a $M/G/K$ queuing system. In each CS the incoming EVs are serviced in order of their arrival (FIFO). Charging time is divided into $T$ timeslots each having a duration $\tau$ seconds. These timeslots are indexed by $t$, that is $t \in \{1, 2, ..., T\}$.

#### A. Electric Vehicle Arrivals

As mentioned in Section II the literature typically assumes a steady arrival rate in the models to capture vehicle behavior. This may result in large deviations between simulation and reality [16], [20]. In real life vehicle arrivals in refueling stations (both electric and conventional) are more frequent in some intervals of the day and less frequent in some others. Thus, in our work EV arrivals in CSs are modeled as a Poisson stochastic process, with a variable, i.e. time-dependent, mean arrival rate $\lambda(t)$. As a result the number of EV arrivals during $t$ is $a_N(t) \sim \text{Poiss}(\lambda(t))$.

#### B. Electric Vehicle Model

Each EV $i \in I$, arrives with a state-of-charge $SOC_{in}^i \in [0, 1]$ which is a normally distributed random variable, where a value equal to 1 indicates full battery and a value equal to 0 indicates empty battery. Its battery capacity is modeled by a discrete random variable $B$ in kWh that follows the distribution shown in Figure 1. This probability mass function results from the market shares each EV type occupies as mentioned in [13]. The Electric Efficiency (EE) per Kilometer is presented in Table I and was extracted from [21], which contained the latest data on EE of All-Electric Vehicles as of 2018. Finally, every EV knows the information about the distance of its next trip. This is modeled as a continuous normally distributed random variable $T_i \in [0, 120]$ given in $Km$.

#### C. Charging Station Model

Every CS has a Central Control Unit (CCU). This CCU monitors the charging procedure in every charger and is responsible for gathering the charging information from the recently arrived EVs as described in Section III-B. Every charger has its charging rate $L_k$ and a queue $Q_k(t)$ which represents the amount of energy the charger has to deliver to the EV charging in time $t$. The charging rate depends on the type of the charger [1]. The charger types we use in our model are...
summarized in Table II. Also a binary variable $x_k(t) \in \{0, 1\}$ is used to indicate whether the $k_{th}$ charger is available at time $t$ ($x_k(t) = 0$), or not ($x_k(t) = 1$). Finally each CS has a cumulative queue $U_n(t)$ for unallocated EVs, which in turn represents the amount of energy the charging station has to give to the EVs currently waiting to be charged.

<table>
<thead>
<tr>
<th>Charger Type</th>
<th>Charging Rate (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level-1</td>
<td>5</td>
</tr>
<tr>
<td>Level-2</td>
<td>15</td>
</tr>
<tr>
<td>Level-3</td>
<td>30</td>
</tr>
<tr>
<td>Tesla Super Charger</td>
<td>&lt;200</td>
</tr>
</tbody>
</table>

**Table II**

**TYPES OF CHARGERS**

**D. EV Allocation**

Every CS has an Allocation Matrix $H_N^t$ that contains all $x_K(t)$ variables. Every time slot the CCU checks if there are any arrivals $a(t)$, and whether any charger is available. If there is an available charger, the CCU allocates the $i_{th}$ EV to the $k_{th}$ charger by queuing its battery requirement $R_i^{in} = SOC_i^t + B_i$ into the $Q_k$. If there is no charger available the CCU queues $R_i$ into queue $U_n$. The allocation procedure is explained in detail in Algorithm 1.

**IV. PROPOSED ALGORITHMS**

Having clarified our system model, we will now formulate our optimization problem. First we define the charging time to be the sum of the time an EV waits in the queue to be charged $w_i^q$ and the time it takes to charge $w_i^c$.

$$w_i = w_i^q + w_i^c$$  \hspace{1cm} (1)

Also by average waiting time we will be referring to:

$$\bar{w}_i = \frac{\sum_{m=1}^{M} w_i(m)}{M}, m \in \{1, 2, ..., M\}$$  \hspace{1cm} (2)

where $M$ is the set of EVs that have been charged and have left the station.

Our main objective is to minimize the waiting time $w_i$ for each EV $i$. The original problem can be formulated as follows:

$$\min \sum_{i \in I} w_i$$  \hspace{1cm} (3)

subject to

$$R_i^{in} \leq B_i \quad \forall i \in I$$  \hspace{1cm} (4a)

$$R_i^{in} \leq w_i^c + L_k \leq (B_i - R_i^{in})$$  \hspace{1cm} (4b)

$$\lim_{t \rightarrow \infty} \frac{\sum_{n=1}^{N} (U_n + \sum_{k=1}^{K} Q_k)}{t} = 0$$  \hspace{1cm} (4c)

With constraint (4a) we ensure all EVs will have an initial charging requirement lower than their battery capacity, when entering the system. In constraint (4b) charging time $w_i^c$ is bound to a maximum of a full battery charge $B_i$. Finally constraint (4c) ensures queue stability of the system [5].

The queuing dynamics of the system are defined as:

$$Q(\tau + 1) = Q(\tau) + a(\tau) - l(\tau)$$  \hspace{1cm} (5)

Where $Q(\tau + 1)$ and $Q(\tau)$ are the charging queue backlogs in respective time slots, $a(\tau)$ is the new energy demand and $l(\tau)$ is the energy demand that was satisfied during the current time slot. As a consequence the above constraint (4c) can be fulfilled only when $a(\tau) \approx l(\tau)$. However, this will result in some EVs having to stop charging and leave the system even though they do not have enough battery charge to reach another CS.
In order to issue with the inequity described above, we propose two algorithms that both minimize the waiting time but at the same time do not force EVs out of the system in a way that is sub-optimal for them and the system overall.

### A. Adaptive Percentage Charging Algorithm (APCA)

In this first algorithm we modify our optimization problem (3) - (4c) so that the CCU adjusts the charging percentage up to which every EV charges, if the current charging queues become unstable. To achieve that we introduce a constraint variable \( p_i(\tau) \). This constraint variable has an initial value equal to one \((p_i(\tau) = 1)\) when EV \( i \) enters the system at time \( \tau \), and changes according to the charging queue backlog.

So our optimization problem now is reformulated as follows:

\[
\min \sum_{i \in I} w_i \quad \text{(6)}
\]

subject to

\[
R_{in}^i \leq B_i \quad \forall i \in I \quad \text{(9a)}
\]
\[
R_{in}^i \leq w_i^* L_k \leq p_i(t) \cdot (B_i - R_{in}^i) \quad \text{(9b)}
\]
\[
\sum_{n=1}^{N} (U_n + \sum_{k=1}^{K} Q_k) \lim_{t \to \infty} \frac{\sum_{n=1}^{N} (U_n + \sum_{k=1}^{K} Q_k)}{t} = 0 \quad \text{(9c)}
\]
\[
p_i(t) \pm dp \in [0, 1] \quad \text{(9d)}
\]

When the current queuing time becomes greater than the average queuing time then the charging percentage drops by \( dp \). Respectively when the current queuing time is smaller than the average queuing time, charging percentage grows by \( dp \). The APCA is explained in detail in Algorithm 2.

#### Algorithm 2 Adaptive Percentage Charging Algorithm

**Input:** \( w_i^q, Q_k(t), U_N(t) \)

**Output:** \( p_i(t) \)

1: \( \text{if } \sum_{i=1}^{I} w_i^q > \overline{w} \text{ then} \)

2: \( p_i(t) = p_i(t-1) - dp \)

3: \( \text{else if } \sum_{i=1}^{I} w_i^q < \overline{w} \text{ then} \)

4: \( p_i(t) = p_i(t-1) + dp \)

5: \( \text{end if} \)

6: \( \text{return } p_i(t) \)

### B. Adaptive Next Trip Charging Algorithm (ANTCA)

In the second algorithm we propose, when the CCU detects queue instability, it changes its charging policy, so that EVs are charged sufficiently enough to reach their next destination. Our optimization problem is again slightly modified as shown in Eq. (8) - (9e) next:

\[
\min \sum_{i \in I} w_i \quad \text{(8)}
\]

subject to

\[
R_{in}^i \leq B_i \quad \forall i \in I \quad \text{(9a)}
\]
\[
R_{in}^i \leq w_i^* L_k \leq p_i(t) \cdot (B_i - R_{in}^i) \quad \text{(9b)}
\]
\[
\sum_{n=1}^{N} (U_n + \sum_{k=1}^{K} Q_k) \lim_{t \to \infty} \frac{\sum_{n=1}^{N} (U_n + \sum_{k=1}^{K} Q_k)}{t} = 0 \quad \text{(9c)}
\]

As in Algorithm 2, when waiting time exceeds the queue average waiting time, the charging station charges EVs sufficiently enough to reach their next destination, based on the information they provide about the distance they have to drive. When the current queue waiting time does not exceed the queue average waiting time EVs are charged to maximum percentage. The ANTCA is described in in Algorithm 3.

#### Algorithm 3 Adaptive Next Trip Charging Algorithm

**Input:** \( w_i^q(t), Q_k(t), U_N(t) \)

**Output:** \( p_i(t) \)

1: \( \text{if } \sum_{i=1}^{I} w_i^q > \overline{w} \text{ then} \)

2: \( p_i(t) = T_i \cdot EE_i / B_i \)

3: \( \text{else if } \sum_{i=1}^{I} w_i^q < \overline{w} \text{ then} \)

4: \( p_i(t) = 1 \)

5: \( \text{end if} \)

6: \( \text{return } p_i(t) \)

### C. Admission Control Algorithm (ACA)

We designed an Admission Control Algorithm (ACA) that minimizes charging demands in CSs, when EV arrivals increase. Specifically with ACA, CSs have the option of rejecting some EVs whose charging demands are smaller than those of other EVs. Let us assume, for example, that a CS is currently charging EVs at a maximum of 60% EVs. Let us assume, for example, that a CS is currently charging a fraction of its EVs such as \( p_{in} \). However in the proposed charging policies the queue stability issue is also taken into account.

### V. Performance Evaluation

Our simulation is conducted in an urban environment in a full 24-hour cycle. During this cycle there are periods with difference in frequency of arrivals as can be seen in Figure 2. We consider CSs that have three chargers, two of them are...
Algorithm 4 Admission Control Algorithm

Input: $Q_k, U_N, p_i(t), SOC_i^{in}, B_i$
Output: $H_N^i$
1: if $p_i(t) > SOC_i^{in}$ then
2: Allocate EV per Algorithm 1
3: end if
4: return $H_N^i$

Level-2 and one is Level-3. The total EVs entering each CS are average to 450. This happens because we opted for an arrival process which is of stochastic nature.

Our proposed algorithms APCA and ANTCA are put to an initial comparison with the First-Come-First-Serve (FCFS) algorithm which is the basic serving algorithm of Queuing Systems in order to identify their applicability to more complicated Queuing Systems. The ACA method, which is a kind of smart charging technique, is also applied to both of the proposed algorithms with positive effects in the minimization of the waiting time.

A. APCA & ANTCA Queue Congestion Evaluation

Here we evaluate the effect the proposed algorithms have on the system. As we can see in Figure 3 with ACPA we achieve almost a 25% reduction in waiting time over FCFS algorithm. However, there is still room for improvement, something that ANTCA achieves, practically eliminating waiting time when queues are unstable.

Concerning the average queue backlog we can see in Figure 4 that the results were similar as above. In FCFS it is expected that the average queue backlog will keep growing as no EV is leaving until it is fully charged. With APCA we see that, when the queue starts growing so that the system becomes unstable, some EVs leave the system because they are charged not at 100% as in FCFS but at a lower percentage. Finally ANTCA is the algorithm that burdens the least the CS as when it detects instability starts charging EVs sufficiently enough for their next trip.

We observe the same behavior in the number of total EVs in the system as seen in Figure 5. It is worth noticing that both APCA and ANTCA have almost identical behavior concerning both Average Backlog and Total EVs in the system as they have the same notion of a stable system embedded in them.

As mentioned in Section IV-C, another way to decongest CSs is via ACA. The results, shown in Figure 6, reveal an obvious reduction of the average waiting time EVs experience for APCA and in Figure 7 for ANTCA.

Table III provides a summary on the evaluation of our algorithms. It can be seen that neither of the proposed algorithms is superior in every way to the others. APCA does offer a higher $SOC^{out}$ but it charges 20% less EVs than ANTCA in the same time period. On the other hand, we see that the application of ACA on both the proposed algorithms does reduce the load from the CSs but does not increase $SOC^{out}$ dramatically. This
TABLE III
ALGORITHM EVALUATION SUMMARY

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>FCFS</th>
<th>APCA</th>
<th>APCA+ACA</th>
<th>ANTCA</th>
<th>ANTCA+ACA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rejection Probability</td>
<td>0</td>
<td>–</td>
<td>0.75</td>
<td>–</td>
<td>0.7</td>
</tr>
<tr>
<td>EVs Charged (%)</td>
<td>14</td>
<td>40</td>
<td>25</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Mean $SOC^{out}$</td>
<td>1</td>
<td>0.58</td>
<td>0.60</td>
<td>0.48</td>
<td>0.50</td>
</tr>
</tbody>
</table>

can be explained by the fact that EVs that are not rejected have such a low $SOC^{out}$, that it does not suffice for their next trip.

VI. CONCLUSION

In this work, we proposed two adaptive algorithms for optimizing the waiting time of EVs in charging stations. Our proposed scheme not only does not use a-priori knowledge of vehicle arrivals, but also two adaptive charging algorithms are proposed, both of them relying on queue stability of CSs, without need for infrastructure cost or wireless communication overhead between the electric vehicles. The proposed algorithms and our simulation results in Section V prove that both algorithms can be applied in charging stations located in urban environments, with a fraction of charging stations adopting APCA and another fraction of them ANTCA. We believe that in combination those two algorithms can handle charging rush hours, without any modification to the charging stations or the power grid. A detailed exploration of such a combined approach is part of our future work. Furthermore, we also plan to evaluate our approach over networks of CSs, as part of our future work.

REFERENCES


