Abstract—We consider a wireless sensor network (WSN) where each sensor node samples a random signal and transmits the result to an access point (AP)/fusion center (FC). The WSN operates under the sub-1GHz IEEE 802.11ah MAC/PHY standard and it also includes a relay node that may be used for forwarding the data to the FC. The FC collects the data from the sensors and the relay and estimates the random signal of each sensor with the linear weighted least squares (WLS) algorithm. The objective of the FC is to minimize the power consumption of the sensors subject to an MSE distortion constraint by selecting the use of the relay for the sensors that need it. We cast the power minimization problem as a mixed integer linear program (MILP).

Our detailed simulation results indicate the efficacy of our scheme under a path loss channel model and randomly deployed sensor populations, while the performance gains are increased even more for denser sensor population. Since the IEEE 802.11ah standard leaves open aspects related to the relay selection process for IEEE 802.11ah-based WSNs, our system can be readily implemented in this emerging class of WSNs.

Index Terms—Wireless sensor networks, cooperative systems, parameter estimation, optimization, Internet of Things, Smart Grid, IEEE 802.11ah.

I. INTRODUCTION

The basic task of any wireless sensor deployed for an Internet of Things (IoT) application is to measure a parameter of interest and transmit it through a wireless link to an access point for storage or further processing. When multiple sensors collect data, then the observations collected from this wireless sensor network (WSN) are processed by a fusion center (FC) that combines them to improve the estimation accuracy of the random signal. In these WSNs one of the most well-known problems is their power limitation since they are usually deployed without connection to a grid-connected power source. This creates a significant problem since a lower transmission power also minimizes the probability of correct packet decoding at the FC. Eventually, with fewer observations of the signal at the FC, the mean square error (MSE) distortion of the estimation is increased leading to poor quality estimates [1].

In recent years several new problems have been adding up for WSNs and are generated by real-life requirements and also technical novelties of newer standards. The first significant problem is that the increasing adoption of IoT applications means that the population of sensors that are present within a specific geographic area is increased rapidly. When a sub-1GHz physical layer (PHY) is adopted like in the emerging WSN standard IEEE 802.11ah [2], the transmission range of the wireless device is extended to more than a kilometer even with transmission power levels that are well within the limit posed by regulating agencies. The two previous observations mean that the number of sensors that a single access point (AP) is expected to serve will be typically very high (can be more than 6000 in IEEE 802.11ah [2]). The performance implications of the previous situation is that the high number of nodes means higher contention for the medium and so fewer bandwidth resources. Eventually the sensor node has to use higher transmission power to ensure that the fewer transmission opportunities that it has are received at the FC. Otherwise the required MSE may not be achieved.

A second problem is that their is an absence of data correlation across independent sensor measurements, and consequently no smart techniques like in-network processing [3] can be used to reduce the communication bandwidth. This is the case for example is smart monitoring applications (electricity, gas, water, home, health, building, e.t.c.) where the parameters of an autonomous sensor are collected and are typically independent of the neighboring devices. Hence, all the data from a sensor have to be communicated and they can only be locally processed.

Given the previous problems, i.e., a large population of WSN nodes have to communicate a wealth of uncorrelated data, the question is how to achieve a prescribed level of MSE for the signal measured by each sensor, without requiring from the sensors to use excessive power levels. More specifically, in this paper we are concerned with the problem of minimizing the power of the IEEE 802.11ah-based WSN when several independent random parameters are estimated. Our system model that is illustrated in Fig. 1 considers also the use of a relay for data forwarding to the AP [2]. The intuition is that the introduction of a relay can improve the reliability of wireless transmission, limiting thus the need for additional bandwidth, channel accesses, and of course transmission power from the sensors. This relay-based topology has recently been adopted.
in the emerging IEEE 802.11ah standard. However, the decision where a relay is actually used by a sensor is an aspect currently not specified in the standard and is open to the system implementation. Overall, our framework can be used as a tool that can allow the operator to configure the IEEE 802.11ah-based WSN in a power-efficient manner by deploying relays.

**Related Work.** Power consumption and its interplay with the estimation accuracy in WSNs has attracted significant research efforts. Linear distributed estimation of correlated data under power or MSE constraints has been studied thoroughly in the literature [4] and in several subsequent works after that. In [5] the authors added the element of noisy fusion center and they designed optimal pre-coding matrices (filters) for minimizing MSE. In [6] the authors proposed an opportunistic protocol for the problem of power allocation and sensor selection under an MSE constraint when the sensors were interconnected through a star topology. The problem of MSE distortion reduction for Gaussian sources under power and rate constraints was studied in [7]. Sensor selection has also been thoroughly considered with the objective of power minimization in [8]. However, all the previous works did not consider node cooperation through relays. The potential to use relays was studied in [9]. In that work the authors studied two-hop multi-sensor relay strategies that minimize the MSE subject to either local or global power constraints. That work considered a relay network with several relays, one sensor, and one destination. More recently, in [10] the authors proposed a power allocation framework for an estimation problem similar to the one defined in this paper. Relay nodes were introduced to form complex branch, tree, and linear topologies. In the most generic topology the authors considered that sensor nodes were assigned statically to relay nodes forming thus a well-structured hierarchical tree topology.

The concrete contributions of this paper with respect to the related work are the following:

- A cross-layer MSE distortion model for a linear estimation algorithm that unlike related work takes into account both the impact of channel contention in IEEE 802.11ah, and the use of cooperative relay-based transmissions.
- An optimization framework for power-optimized linear estimation in a WSN that operates under IEEE 802.11ah, and an associated low complexity solution algorithm.

**II. SYSTEM MODEL AND ASSUMPTIONS**

**Network Model and Objective.** Our objective is to estimate uncorrelated random signals, compactly described as the vector \( \mathbf{\theta} = [\theta_1 \ldots \theta_N]^T \), through a WSN of \( N \) nodes that belong in the set \( \mathcal{N} \). For signal \( \theta_i \) collected at sensor \( i \) our goal is to ensure that the MSE is not more than \( d_i^{\text{MAX}} \). We assume that the observations are uncorrelated since the sensors sample independent data in a large geographic area (Fig. 1). There is a FC, and also a single relay node deployed within the sensor population. The relay may be used by a sensor to meet the desired MSE. Note that this WSN model may represent a part of a larger deployment of sensors that may be similarly distributed and use local relays.

**Sampling and Quantization.** Each sensor collects several observations (samples) during a period of \( T \) seconds that depends on the monitored phenomenon. The input signal to the quantizer of sensor \( i \) is an analog sample \( \theta_i + z_i \) that consists of the data and the AWGN sampling noise with \( z_i \sim \mathcal{N}(0, \sigma_{z_i}^2) \). After quantization the signal is:

\[
y_i = Q(\theta_i + z_i) = \theta_i + z_i + q_i
\]

(1)

The quantization noise \( q_i \) across sensors is independent. From (1) we see that the variance of the quantized signal at sensor \( i \) is:

\[
\sigma_{q_i}^2 = \sigma_{\theta_i}^2 + \sigma_{z_i}^2 + \sigma_{q_i}^2
\]

For a quantization with \( R_i \) bits/sample, the variance of the quantization noise (or the distortion), under the use of a uniform probabilistic quantizer \( Q(\cdot) \) [11] at sensor \( i \) is:

\[
\sigma_{q_i}^2 = \frac{A^2}{(2^{R_i} - 1)^2}
\]

(2)

In the above, \( 2A \) is the range of the sensed signal.

**Modulation and Transmission.** We assume that the sensor collects \( K \) source samples of the form given in (1) and creates a packet of \( K R_i \) bits. Each sensor \( i \) uses a modulation and coding scheme (MCS) scheme that is characterized by a spectral efficiency of \( R_i \) bits/symbol. This parameter takes into account the use of a capacity-achieving AWGN code. The mapping of the quantized digital samples to digital baseband symbols after channel coding and digital modulation is denoted as follows:

\[
x_{d_i} = \text{CC-PSK}(y_i)
\]

(3)

The transmission of \( x_{d_i} \) takes place over a wireless link \( h \) with flat quasi-static Rayleigh fading. The average channel gain is affected by the path loss (distance between the sensors, the FC, and the relay) and its precise average value will be defined in our performance evaluation section. Also \( P_i \) is the transmit power at sensor \( i \). Based on our previous discussion, the baseband representation of the received signal at the FC from sensor \( i \) is:

\[
y_{i,fc} = \sqrt{P_i} x_{d_i} h_{i,fc} + w_{fc},
\]

(4)

with \( \sigma_{w_{fc}}^2 = 1, \forall i \in \mathcal{N} \), and \( w_{fc} \sim \mathcal{N}(0, \sigma^2) \) is the receiver noise. The signal transmitted from the source to the relay and the relay to the FC can be written similarly. For the decoded signal of sensor \( i \) the relay uses power \( P_r \).

**IEEE 802.11ah.** To transmit the digital packet to the FC each node must access the channel. It does so with the IEEE 802.11ah [2], for which we model its core PHY/MAC functionalities in our overall system model. Regarding the 802.11ah PHY features, it uses the lower MCSs of IEEE 802.11ac. BPSK with code rate \( \frac{1}{2} \) was adopted to ensure long range and this was the selected to be the value for \( R_i \). At the
MAC the standard uses the distributed coordination function (DCF) for contenting for channel access [2].

**Relay Functionality.** Once a node obtains access to the channel it transmits the packet in one slot and in subsequent slot it may use the relay. The role of the relay is simple since it only decodes and forwards (DF) the digital transmitted data. This type of relay functionality is specified in the IEEE 802.11ah standard [2].

**Paper Notation.** Regarding the paper notation matrices are denoted with bold capital letters, i.e. A. Bold lowercase denote vectors. The matrices $A^T$, $A^H$, $A^*$, are the transpose, Hermitian, and conjugate of A. Also $\text{Tr}(\cdot)$ is the trace of a matrix.

### III. CROSS-LAYER DISTORTION MODEL FOR COOPERATIVE SENSOR DATA TRANSMISSION WITH IEEE 802.11AH

#### A. Cooperative Decode-and-Forward Protocol Model

To reach our final goal, which is to identify whether the use of the relay is needed for achieving a certain distortion threshold, we have to model first the performance of the cooperative DF protocol among other aspects of our system. To do that we have to calculate the probability that a transmission with this protocol fails. We follow an information-theory driven approach. In particular, since the communication channel is quasi-static Rayleigh fading, the reception of a packet cannot be guaranteed with probability 1. Hence, the so-called outage probability of this cooperative transmission must be calculated that indicates the average number of decoded packets [12]. The outage probability for a point-to-point link between sensor i and FC is

$$P_{out,i} = \Pr\{\log_2(1 + \frac{P_i|h_{i,fc}|^2}{\sigma^2}) \leq R_t\} = 1 - \exp(\frac{-\left(\frac{2R_t}{\sigma^2} - 1\right)}{P_i E|\bar{h}_{i,fc}|^2/\sigma^2}),$$  

(5)

where $\sigma^2$ is the variance of the AWGN noise at the receiver. The closed-form result is because $|h_{i,fc}|$ is Rayleigh distributed. Similar expressions apply for all the point-to-point links. Hence, the end-to-end outage probability with our protocol is:

$$P_{out}(i, P_i, P_r) = P_{out,i} + P_{out,r}(1 - P_{out,i,r})$$  

(6)

The above is true because the direct transmission from the sensor to the FC and the transmission from the relay are independent, and also because if the transmission from the sensor to the relay fails, then the cooperative transmission along that path fails completely. By combining the different versions of (5) and also (6), we obtain the final expression for the end-to-end outage probability that depends on several parameters.

Even though in our outage expression only few parameters are involved, the resulting MSE of our estimation algorithm depends on other parameters of our network model. As we will see in the next paragraph in detail, the MSE distortion performance depends on the number of available observations collected over a period $T$.

#### B. Modeling the Number of Observations

Before we move on and calculate the MSE of the estimation algorithm used at the FC, we have to calculate the average number of observations that the algorithm receives in a specific time period. This number will affect directly the MSE while the precise analytical formula will be derived in the next section. However, calculating the outage probability in (6) is not enough. A sensor operating under the DCF mode in IEEE 802.11ah will share the channel with the remaining nodes. To calculate the achievable throughput we use the model in [13] that has also been validated in practice, and also considers unsaturated traffic input at each node. More specifically, with MCS $R_t$, a packet of $KR_t$ bits that is produced every $T$ sec, and $N$ contending nodes we denote the throughput as $S_{DCF}(R_t, K, N, T)$ (equation 10 in [13]). Other 802.11-specific timing parameters can be found in [13]. Note that the previous model considers only the impact of collisions in the throughput. However, unlike the related work we consider packet losses due to errors according to (6). The previous discussion leads to the following expression for the average number of observations available at the FC sensor i:

$$M_{rec}(i) = S_{DCF}(R_t, K, N, T) \cdot (1 - P_{out}(i, P_i, P_r)) \cdot \frac{T}{R_t}$$  

(7)

The last part of the formula above is very important since we multiply with the period $T$ (sec) that we exercise the estimation, and we divide with $R_t$ (bits/observation). Note that we did not model any retransmission scheme, while this can be accomplished methodologies such as the one found in [14]. Hence, the final result is the average number of samples that we receive over these $T$ seconds.

#### C. MSE Distortion Model

At this point we have calculated how many observations of the random signal $\theta_i$ the FC has received by taking into account the compression/quantization at the source, the use of cooperative relay-based communication in (6), and the use of DCF channel access from IEEE 802.11ah MAC in (7). The final step is to convert (7) to the actual MSE distortion.

In our system we assume no knowledge of the prior data distribution and so we use the weighted least squares (WLS) linear estimator which is the best linear unbiased estimator (BLUE) for the Bayesian linear data model we consider in this paper [1]. Recall also that the data from the different sensors are uncorrelated because the sensors are located at large physical distances. This last assumption means that the distortion of the estimated data for one sensor will be independent from the others. Hence, the estimation accuracy for the random signal monitored by one sensor will be improved only by obtaining more measurements from that sensor.

To develop our model let us denote with $c=1$, a $1 \times M_{rec}(i)$ matrix that expresses the number of available $M_{rec}(i)$ observations after decoding in the end of the period $T$. Hence, the Bayesian linear data model for the data of one independent sensor is

$$y_i = c\theta_i + z_i + q_i,$$

(8)
where \( v_i \) is the combined AWGN sampling and quantization noise for several observations of our signal \( \theta_i \). Let us now consider the WLS estimator according to our previous data model [1]:

\[
\hat{\theta}_{i,\text{WLS}} = (c^H \Sigma_{v_i}^{-1} c)^{-1} c^H \Sigma_{v_i}^{-1} y_i
\]

(9)

To calculate the MSE, first we calculate the covariance matrix of the error of the WLS estimator. Formally, we know that the covariance matrix for the error of the WLS estimator is [1]

\[
\Sigma_{e,i} = (c^H \Sigma_{v_i}^{-1} c)^{-1},
\]

(10)

where each diagonal element in the matrix \( \Sigma_{v_i} \) can be easily shown to have the same value equal to:

\[
[\Sigma_{v_i}]_{k,k} = \sigma_i^2 + \sigma_{q_i}^2
\]

(11)

But in this case the error matrix in (10) is actually a scalar since we estimate a single variable. Hence, the total MSE from using \( M_{rec}(i) \) measurements is the trace of the above error matrix in (10). Thus, it is:

\[
\text{MSE}(i) = \text{Tr}(\Sigma_{e,i}) = \left( \sum_{k=1}^{M_{rec}(i)} \frac{1}{\sigma_i^2 + \sigma_{q_i}^2} \right)^{-1} = \frac{\sigma_i^2 + \sigma_{q_i}^2}{M_{rec}(i)}
\]

(12)

This last expression is combined with (7) to obtain the final closed-form MSE result.

IV. OPTIMIZATION FORMULATIONS

We consider the MSE expression obtained from our analysis as one of the constraints to our optimization problem while the power consumption is the optimization objective. We desire to minimize the sensor power which means that it might be possible that the use of the relay is not needed since the additional MSE improvement is marginal and the MSE constraint can be satisfied without extra power consumption. To proceed with the formal definition of our problem we define \( x_i \) as a binary variable that indicates whether sensor \( i \) transmits through the relay. Thus, the optimization variables for the problem we define are \( P_i \) and \( x_i \), and in vector form \( x = (x_i \in \{0,1\} : i \in \{1,\ldots,N\}, P = (0 \leq P_i \leq P_{\text{MAX}} : i \in \{1,\ldots,N\}) \). The objective is to minimize the sum of the power consumed at the sensors and the relay subject to the same individual MSE constraints:

\[
\min_{P,x} \sum_{i=1}^{N} (P_i + x_i P_{\text{r}})
\]

subject to

\[
\frac{\sigma_i^2 + \sigma_{q_i}^2}{M_{\text{rec}}(i)} \leq d_{i}^\text{MAX}, \forall i \in N \quad (C1)
\]

\[
0 \leq P_i \leq P_{\text{MAX}}, \forall i \in N \quad (C2)
\]

Note in the above formulation that there is no need for a constraint that ensures that the relay is used by only one sensor. The reason is that the IEEE 802.11ah MAC ensures orthogonal access.

A. Solution and Implementation

Regarding the sensor parameters that are needed for solving the problem note that most important ones are the average channel between the sensor, the FC, and the relay. This means that they can be estimated at the FC from measurements from these devices during long time periods. The variance of the AWGN sampling noise can similarly be available at the FC since it is device-specific. This means that the problem can be solved at the FC without the need for explicitly communicating information from the sensors. Now the problem formulation clearly constitutes a non-convex mixed integer non-linear programs (MINLP). However, it can be decoupled across the sensors since there is no coupling constraint. The algorithm we follow for solving the problem is to set first \( P_r = 0 \) and start increasing in steps \( P_i \) until the desired MSE threshold is met. We also set \( P_r \neq 0 \) and follow the same procedure for increasing \( P_r \). From the two solutions we select the one that gives the lower \( P_i + P_r \). This is communicated to the sensor.

V. PERFORMANCE EVALUATION

We present simulation results for evaluating the performance of our approach. The system parameters are set as follows \( \sigma_i^2 = 1, \forall i \in N; \sigma^2 = 10^{-4}, R_i = 8 \text{ bits/source sample}, A = 1 \text{ Volt}. \) We also set \( T = 10 \text{ seconds}, \text{i.e.}, we assume that the phenomenon changes every 10 seconds. We assume a path loss channel and so \( E[|h_{i,j}^2|] = 1/d_{\text{norm}}(i,j)^\alpha \), where \( \alpha \) is the path loss exponent set equal to 3, and \( d_{\text{norm}}(i,j) \) is the distance between nodes \( i,j \) normalized in the range from 0 to 1Km. The timing parameters of the IEEE 802.11ah MAC were obtained from the current standard [2]. Also, we explained earlier the most robust MCS is selected, i.e., BPSK modulation with a coding rate of \( \frac{1}{2} \). We also evaluate two different flavors of our optimization namely one where the relay is always selected, i.e., it is removed from the objective since we do not consider its power consumption incurs any cost (e.g., it is connected to the power grid). A second system where both the sensor and the relay power are part of the objective as in our main formulation.

A. Results for a Single Sensor

For this first scenario we evaluate the performance of a single sensor since the solution for this case is the fundamental building block of our complete optimization. At a distance 0.5Km from the FC we place a relay. In the straight line defined between the FC and the relay, we check different sensor locations: The sensor locations in the left of the \( x \) axis are close to the FC while when we move to the right the sensor approaches the relay (located at 0.5), and after that point it moves away from both of these devices.

In Fig. 3(a) the results for the case that relay is always used and their is no cost in doing so. When the sensor approaches very close to the relay, then the required power from the sensor is reduced considerably, leading to this “dip” in the power consumption, due to the fact that the sensor has to use minimal power for reaching the relay. This might also be a realistic situation since these relay devices may be connected to the power grid. Regarding the optimal results when the relay may not always be used, since its power consumption is part of the objective, they are presented in Fig. 3(b). When the relay power is \( P_r = 0.5 P_{\text{MAX}} \) its presence is more useful at relatively small distance of the sensor from the FC, i.e., in the range between 100 and 300 meters. However, after 300 meters it
is more costly to use the relay with $P_r=0.5P_{\text{MAX}}$, and more efficient to use a relay with $P_r=0.3P_{\text{MAX}}$.

Both these figures provide concrete guidelines with respect to the strategy that the sensor has to follow, and this has clear dependence on whether the relay power is used in the objective. That is, in the first scenario nodes close to the relay should strive to use it as much as possible regardless of the relay power. In the second scenario, sensors should also try to use the relay, but the optimal relay power should be set depending on the relative distance of the nodes as illustrated in Fig. 3(b).

B. Results for Multiple Sensors

For a large network we create random WSN instances and we average the results. The sensors are spread randomly and uniformly in a circle. In the middle of the circle there is the relay, while in the edge there is the FC. Then, we solve the optimization problem by using as inputs the average channel gains. Subsequently, we configure each sensor with the optimal power setting, and we simulate the Rayleigh channel.

Due to the random placement of sensor nodes a subset of them uses the relay and another subset it does not according to the result of our optimization. When the relay is always used, in the results of Fig. 3(c) we notice significant reduction in the sensor transmission power for every configuration of the relay power. The reason is that the cost of using the relay is zero in this case and the cooperative transmission is always the preferred choice. Another important result is that as the sensor population is increased, we have sharper increase in transmission power when the power of the relay is low (e.g., for $P_r=0.01P_{\text{MAX}}$ and for 70 sensors or more the increase is worse than linear). The reason is that the
increased sensor density increases the contention in the IEEE 802.11ah MAC. To meet the distortion constraint, since the effective communication rate is lower according to (7), higher transmission power is required to minimize $P_{out}$. However, for higher relay power the phenomenon is not so significant since the relay can compensate for this behavior and effectively shift the sharper increase in the transmission power for higher node densities. For example for $P_r = 0.1P_{MAX}$ the increase starts from 90 nodes. However, if the required MSE is equal to 0.5, then we can afford to have a higher Pout and so lower transmission power can be used.

In the scenario that the relay selection is applied with our full-fledged system, the related results are illustrated in Fig. 3(d) and we present now the total power (i.e., $P_i + P_r$). We observe smaller differences in this case since the relay incurs a cost and hence it is used less frequently. For higher relay power the cost of using is even more important which means that not so significant additional benefits can be obtained (consistently with our results for one sensor). It is also important to see that unlike the previous paragraph, allowing a more relaxed MSE threshold does not help significantly since a higher Pout does not lead to more aggressive use of the relay.

VI. CONCLUSIONS

In this paper we presented a cross-layer model and an optimization framework for minimizing the power consumption of an IEEE 802.11ah relay-based WSN that executes a distributed estimation application. Our framework allows the derivation of the optimal power settings independently for each sensor as a function of its relative location towards the FC and the relay. The performance results indicate that deploying such a relay, even in a randomly deployed sensor population, can lower significantly the power consumption regardless of whether the use of the relay itself incurs a power cost or not. Our future work will include the investigation of different topologies with larger sensor populations.

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REFERENCES