Online Cross-Layer Network Coding in Wireless Ad Hoc Networks

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Abstract—In this paper, we present a cross-layer framework for online optimization of opportunistic network coding in wireless multi-hop networks. Initially we focus on modeling the expected network-coded throughput individually for each wireless station as a function of lower layer parameters like the maximum number of link-layer retransmissions and the transmission mode at the physical layer (PHY). Based on this analysis, we develop a network coding algorithm that optimizes opportunistically and dynamically the expected information content of individual packet transmissions only within a single hop. Subsequently, we devise a distributed cooperation algorithm that allows nodes to select the optimal PHY transmission mode by considering the PHY selection of their neighbors. Our approach can yield significant gains in terms of throughput without employing complex link scheduling algorithms.

I. INTRODUCTION

Network coding suggests that network nodes not only route packets but also execute algebraic coding operations on them [1]. In wireless networks, network coding was originally combined with wireless broadcast to increase the information content for each broadcast transmission [2]. In Fig. 1 we demonstrate the basic idea according to which a node B broadcasts the coded packet \( a \oplus b \) that will result in the decoding of both packets \( a \) and \( b \) at C and A respectively. This simple idea has been applied in multi-hop wireless local area networks and throughput benefits in the order of 3−4 times over the baseline 802.11 have been demonstrated [2].

However, the performance of network coding will be influenced by existing algorithms that are typically used in the lower layers of the protocol stack. For example wireless stations make use of the auto-rate mechanism that is responsible for selecting a PHY transmission mode that is optimal for the current channel conditions [3]. But when network coding is employed a single packet is broadcasted to more than one node which makes the selection of the optimal PHY mode more challenging (see Fig. 1). This problem is identified but not addressed in related works primarily because there is a lack of practical rate adaptation schemes that are suitable for broadcasting [4], [5]. Therefore, there is a need to allocate the channel air-time with criteria that take into account network coding.

The second problem that is particularly important in multi-hop wireless networks is the existence of hidden nodes. If we refer to the topology in Fig. 1, node B suffers from the hidden node problem which means that it will backoff more frequently because of transmissions from nodes E and F. Furthermore, the transmission of coded packets from node B reduces spatial reuse since both nodes A and C must receive the packet with a single broadcast transmission. It has been shown that for this scenario, depending on the traffic requirement at nodes E and F, coding packets in every opportunity is suboptimal [7].

To tackle these issues we develop a cross-layer and cooperative scheme for nodes that employ opportunistic network coding. We use a simple link-layer network coding protocol and from that point we focus entirely on analyzing the performance of network coding jointly with the MAC and PHY layers. The analytical model is used for estimating the expected throughput by considering the available packets in the MAC queue, the level of link-layer automatic repeat request (ARQ), and the selected transmission mode at the physical layer (PHY). Based on the cross-layer performance analysis, we develop an adaptive linear network coding algorithm that opportunistically optimizes the scheduling of broadcast transmissions only within a single hop.

II. NETWORK AND INTERFERENCE MODEL

The network is modeled with a directed graph \( F(\mathcal{N}, \mathcal{V}) \), where \( \mathcal{N} \) and \( \mathcal{V} \) are the set of directional links, and the set of nodes, respectively. We assume that for any link between two nodes there is a counter-part in the opposite direction. Let the number of links in the graph be \( N \). For modeling broadcast transmissions, we adopt the concept of hyper-arcs for referring to the links that are used in single-hop broadcast transmissions similar to [4]. According to this notation, a hyper-arc \( (s, D) \) denotes a set of broadcast packet transmissions from node \( s \) to all the nodes in set \( D \) with \( D \subseteq \mathcal{V} \). We also denote by \( D_k \) a specific broadcast transmission that is a subset of the hyper-arc \( D \), and with \( \mathcal{D} \) all the possible broadcast realizations for a particular node. For example for node B in Fig. 1,
**D** = \{(A, C), (AC)\}. This means that one possible broadcast realization is for example \(D = (AC)\) (i.e. the transmission of a coded packet both to nodes A and C with a single broadcast transmission). In this case then the specific broadcast transmissions are \(D_1 = AC\) and \(D_2 = \emptyset\). Finally, let also \(x_e\) be the air-time that link \(e\) is active, with \(e \in N\).

The final important aspect of the general model we have to built is the conflict graph of \(F\) that contains the interferring relationships among the \(N\) links in the network. The conflict graph can be partially constructed by each node as follows. We assume that the clear channel assessment (CCA) sensitivity is also the minimal interfering signal strength that can corrupt a transmission. Then, the interference range \(R_I(s)\) of node \(s\) with transmission power \(P_{tx}\) can be obtained by

\[
R_I(s) = \sqrt{\frac{k}{c}} \frac{P_{tx}}{CCA},
\]

where \(k\) and \(c\) are environmental constants [8]. In this paper we assume that the PHY transmission mode \(r\) can change which means that in this case the \(P_{tx}\) will also change. If the distance between two nodes say A and B is \(dist(A, B)\), the transmission power can be expressed as follows:

\[
P_{tx}(r) = \frac{P_{tx}(r) \times dist(A, B)^k}{c}
\]

In general, two nodes say A and B interfere with each other when the following condition is true:

\[
dist(A, B) \leq \max(R_I(A), R_I(B))
\]

### III. Link-Layer Network Coding

In this subsection we specify the simple protocol that facilitates the transmission of network-coded packets. This protocol is implemented at the link layer of IEEE 802.11 [9]. Its main feature is that it distinguishes the type of MAC service data units (MSDUs) that are transmitted in the network. Three types are defined: **Type-1** the unicast uncoded data packets, **Type-2** unicast uncoded data packets targeted to multiple opportunistic receivers, and **Type-3** are the multicast coded packets targeted to multiple receivers that can decode innovative packets from them. While the transmission of **Type-1** packets is easy to understand, this is not the case for **Type-2** packets. This type of packet is acknowledged through a form of opportunistic acknowledgments (OACKs) in order to indicate that these are not regular data packets that need reliable transmission. Data packets are tagged as **Type-2** when there is the possibility that the packet will be used for network coding by the target node. Potential coding opportunities can be identified when the traffic pattern in the neighborhood is being observed [9].

### IV. Throughput Analysis for a Single Node

The goal of the analysis we present in this section is to derive a closed-form expression for the expected information rate as a function of the group of packets that are XOR-coded together for a single broadcast transmission, the selected common PHY transmission mode, and the maximum number of link-layer retransmissions.

#### A. Packet Success Probability for a Single Coded Packet

If we assume that the transmitting node is named \(s\) and the common PHY transmission mode for the broadcast transmission is \(r\), the probability of a successful PHY frame transmission for the \(d\)-th receiver can be calculated by

\[
P_{tx}^{(s, d)}(r, r) = [1 - P_{e, data}^{(s, d)}(r)][1 - P_{e, acc}^{(d, s)}(r)], \quad d \in D,
\]

where \(P_{e, data}^{(s, d)}\) and \(P_{e, acc}^{(d, s)}\) are the packet erasure probabilities for data packets and acknowledgments respectively, while \(r_2\) is the most robust PHY mode in 802.11a. The packet error probabilities can be calculated as

\[
P_e = 1 - (1 - BER(\gamma, r))^S_t,
\]

where \(S_t\) is the size of the packets in bits, and \(\gamma\) is the instantaneous channel signal-to-noise ratio (SNR). The BER can be calculated depending on the adopted channel model [8].

Now for the broadcast transmission defined by hyper-arc \(D_b\), the probability of successful delivery by all the intended receivers is given by:

\[
P_{tx}^{(s, D_b)}(r, r) = \prod_{d \in D_b} P_{tx}^{(s, d)}(r, r).
\]

Since we consider retransmissions, the probability of successful delivery with \(n_m\)-th truncated ARQ from (6) is:

\[
P_{succ}^{(s, d)}(l, r, n_m) = 1 - \prod_{i=1}^{n_m} [1 - P_{tx}^{(s, d)}(l, r(i))], \quad d \in D.
\]

An important observation from (7) is that the PHY transmission mode \(r\) of the possible retransmissions can also change since the initial packet may be received by a subset of the receivers. Therefore, we use the notation \(r(i)\) to indicate the PHY mode of the \(i\)-th transmission attempt. From the previous calculations, we can move to the next step and derive the conditional probability that the PHY frame is actually received with the \(n\)-th attempt out of the maximum \(n_m\). By combining (4) and (7) we have, for the \(d\)-th receiver:

\[
P_{succ}^{(s, d)}(l, r, n_m) = \frac{P_{succ}^{(s, d)}(l, r(n))}{P_{succ}^{(s, d)}(l, r(n), n_m)} \cdot [1 - P_{succ}^{(s, d)}(l, r, n)].
\]

We can also calculate the probability that with the \(n\)-th transmission the packet is received by all the intended receivers of the broadcast transmission \(D_b\):

\[
P_{succ}^{(s, D_b)}(l, r, n_m) = \prod_{d \in D_b} P_{succ}^{(s, d)}(l, r, n_m).
\]

#### B. Impact of Packet Loss on the Transmission Delay

For calculating the overall delay we proceed as follows. First we consider the backoff procedure. In the 802.11 MAC, the backoff interval is measured in slot units with duration \(t_s\).

In our case we want to calculate \(T_{bkf}(n)\) which is the average duration of the backoff before the \(n\)-th transmission attempt,

1Note that the fact that opportunistic receivers are also possibly receiving a packet \(l\), is not included into the information rate/goodput calculation since these packets do not contain innovative information.
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or equivalently, the \((i - 1)\)-th re-transmission attempt. This is basically directly related to the average value of the contention window \(W\) as follows [10]:

\[
W_{bkf}(n) = W \times t_s
\]  
(10)

Now the average value of the contention window will depend on the average packet loss probability \(P_{tx}\) as follows [10]:

\[
W = \frac{1 - P_{tx} - P_{tx}^2 P_{tx}^{n-1} \text{CW}_{\text{min}}}{1 - 2 P_{tx}},
\]  
(11)

where \(P_{tx}\) was calculated in (4).

In the general case a packet will be considered lost if the transmission itself fails or the acknowledgment fails. In (4) this distinction was included but now we have to properly evaluate what this means for the delay. In the case of a failed acknowledgment for \(n\)-th data packet transmission the node waits for an extended IFS (EIFS) with probability

\[
P_1(n) = \frac{[1 - P_{e,\text{data}}(l, r(n))] P_{e,\text{ack}}(r(n))}{1 - P_{s,\text{tx}}(l, r(n))}.
\]  
(12)

When the data frame transmission fails then the sender experiences an ACK timeout [11] with probability

\[
P_2(n) = \frac{P_{s,\text{tx}}(l, r(n))}{1 - P_{s,\text{tx}}(l, r(n))}.
\]  
(13)

In the 802.11 standard, the sender waits for a short inter-frame space (SIFS) for the arrival of the acknowledgment in order to allow for the hardware to switch into receiving mode [11]. Therefore, the average waiting time, excluding backoff, before the \(n\)-th retransmission can be calculated for each of these two cases and for each receiver \(d\) by [3]:

\[
L_{w,d}(n) = \begin{cases} 
P_2(n - 1) \cdot [T_{SIFS} + T_{ack}(r(n - 1)) + t_s] & \\
+ P_1(n - 1) \cdot [T_{SIFS} + T_{ack}(r(n - 1))] & \\
+ T_{EIFS} 
\end{cases}
\]  
(14)

Now we have to account for the fact that a (re)transmission might be unsuccessful for certain receivers. Therefore, the overall \(L_{w}\), i.e. the worst case delay is defined by the "slowest" receiver. We can calculate this value as follows:

\[
L_{w}(s, D_k)(i) = \max_{d \in D} L_{w,d}(s, D_k)(i).
\]  
(15)

C. Total Delay of Broadcast Transmissions

The transmission delay should also be calculated in the case of a successful 802.11 frame transmission. In our calculations we have to consider additively the total delay that is added given that \(n - 1\) failed transmissions occurred plus the time for the successful one. By using (9), (10), (14), and (15), the average duration for the transmission of a single broadcast transmission \(D_k\) is:

\[
L_{\text{succ}}(l, r) = \sum_{n=1}^{n_m} P(s, D_k)[n|\text{succ}](l, r, n_m)
\]

\[\cdot \left\{ \sum_{i=2}^{n} [L_{w}(s, D_k)(i) + T_{bkf}(i) + T_{data}(l, r_c)] \right\} + T_{bkf}(1) + T_{data}(l, r_1) + T_{SIFS} + T_{ack} + T_{DIFS}\].

The summation term in the second line of the previous equation represents the total time spent because of failed transmissions (that we derived in the previous subsection) while the third line corresponds to the delay of the final successful transmission. The final parameter that has to be calculated is the average transmission time that is consumed when the retransmission limit \(n_m\) is reached and the transmission for at least one receiver fails. We can easily re-write (16) as follows:

\[
L_{f}(s, D_k)(i) = \sum_{i=1}^{n_m} [T_{bkf}(i) + T_{data}(l, r_c) + L_{w}(s, D_k)(i + 1)].
\]  
(17)

D. Throughput

From the above analytical expressions we can now calculate the expected MAC-layer information rate at a single backlogged node given a specific broadcast transmission \(D_k\). The expected information rate is equal to the ratio of the expected delivered data payload (or innovative packets decoded) to the expected transmission time that includes also the failed transmissions:

\[
I(s, D_k) = \frac{\sum_{j \in D_k} P(s, j)(l_k, r, n_m)}{\sum_{j \in D_k} (1 - P(s, j)) L_f(j) + P(s, D_k) L_{\text{succ}}}. 
\]  
(18)

Also the total expected information rate for a particular coding schedule that is defined with hyper-arc \(D\) is given by:

\[
I(s, D) = \sum_{D_k \in D} I(s, D_k)(l_k, r, n_m). 
\]  
(19)

V. OPTIMIZATION ALGORITHMS

A. Network Information Rate Problem

To proceed with the formal definition of the problem let us denote the group of packets that exist in the MAC queue and can be used for coding as \(L\). We overload the hyper-arc notation \(D\) with the term \(L\) so that \(D(L)\) expresses the set of possible broadcast schedules that can be used for the given group of packets. Assume also that \(l_k\) is the coded packet that results from the \(k\)-th broadcast transmission where \(D_k \in D(L)\). Therefore, the cross-layer opportunistic network coding problem can be written as follows:

\[
\max_{L(\text{Queue})} \sum_{D_k \in D(L)} I(s, D_k)(l_k, r, n_m)
\]  
(20)

subject to

\[
\sum_{D_k \in D} x_k \leq \mu_D(\epsilon), \text{ } \sum_{e \in S(j)} \mu_D(\epsilon) \leq 1.
\]
The first important parameter in the above expressions is $x_k$ and denotes the proportion of time that a broadcast transmission $D_k$ from hyper-arc/schedule $D$ is activated. This should be less or equal from its share $\mu_D(e)$. The second constraint expresses the total air-time that the links of all the nodes in the neighborhood are active which cannot be more than one.

B. Cooperative Air-Time Allocation Algorithm

Local decisions on the PHY mode will affect the air-time that is received by nodes in the immediate neighborhood. To this aim in this section we focus on calculating the air-time share that a node must obtain.

With our algorithm a node calculates the cross-layer parameters that maximize the expected information rate $I$ according to (18). In this way the node can derive the locally optimal PHY $r^*$ and schedule $D$. When the optimal PHY $r^*$ is identified, and is different from the current PHY mode $r^{\text{curr}}$, a PHY mode change should be initiated. This is accomplished by notifying all the nodes within its interference range, and by including in the outgoing message the information for the calculated tuples $\text{information rate/PHY mode}$. This step ensures that all the nodes for which a PHY mode change will have an impact are notified. After the initial notification is sent, the node also receives similar information from the nodes with whom they share a specific clique. By collecting this information, the node identifies the PHY rate that maximizes the information rate for all its broadcast transmissions. The advantage of this algorithm is that it requires participation only from the neighbors of each node that are within its maximum interference range. Note also that when a node identifies the optimal PHY rate, it sends a request for approval for the new rate and it applies the selected rate when all the nodes approve it [12], [13]. The important condition is that a node approves a PHY rate change from one of its neighbors, if this change does not reduce its own expected information rate. As a consequence of the previous condition, convergence is established since every node will have the same information based upon it will identify a unique optimal PHY for the node that initiated the request for a PHY change.

C. Network Coding Algorithm

A greedy heuristic algorithm is adopted for solving the linear program in (20) and identify the optimal group of packets that should be coded $\mathcal{L}$. For each candidate combination $\mathcal{L}$ the algorithm calculates the expected $I$ from (18). Each node also measures the packet erasure rate based on the transmission history of each receiver so that they can be used by the analytical model. Therefore, (18) offers a very good estimate of the information rate for a particular group of receivers. Subsequently, the algorithm calculates $I'$ for broadcast schedule $D'$ by removing from $D$ the slowest receiver, in terms of its PHY rate and PER, i.e. $\{D'\} = \{D\} - 1$. If $I'$ is higher than $I$ the algorithm continues this procedure in order to prune suboptimal receivers. Note that when a receiver is pruned from the broadcast group of coded packets, this is accounted as separate regular unicast transmission. If the pruning cannot increase the estimated information rate, a previous valid schedule defined by $D$ is used.

VI. PERFORMANCE EVALUATION

In this section we evaluate the performance of the proposed cross-layer and cooperative network coding system through a simulation tool built in C. We considered a 1000x1000m geographical area for which random network topologies were generated. Furthermore, we considered CBR packet flows between the nodes in the WANET. All nodes are part of a unicast traffic either as a source or a destination. This means that the number of unicast flows is 50% of the number of nodes we tested for a particular experiment. Three protocols were implemented. The first is NC/802.11 and implements the basic network coding features on top of the MAC that include pseudo-broadcast and the transmission of separate receiver reports [6]. The second system named CL-NC considers the cross-layer network coding algorithm without any form of cooperation. The final system we tested is named CLC-NC to indicate that both cross-layer optimization at individual nodes and also cooperative PHY rate selection are employed.

A. Number of Nodes and Hops

Fig. 2 demonstrates the impact of assigning a different number of nodes to the overall network while the packet flows traverse only one or two hops. More specifically, the percentage of single-hop flows that we configured for this experiment is 60%. This distinction is important since packets that are destined directly to their neighbors cannot be coded. Fig. 2(a) presents results for the throughput gain for the 3 systems under test. With NC/802.11 and CL-NC the nodes that code gradually obtain a smaller fraction of the available bandwidth as the network density increases. In this case we see that the cooperative algorithm is extremely useful since it allows nodes to make more efficient use of the available bandwidth despite the additional message passing overhead it introduces. Additional results for the delay are presented in Fig. 2(b) and suggest that more nodes do not necessarily increase the delay for CLC-NC when compared with conventional routing.

The performance increase is more significant as the number of hops that a flow traverses is increased as Fig. 3(a) indicates. This is a result we expect since the offered load can be distributed to more nodes while the coding gain is

![Fig. 2. Simulations results for different number of nodes in the wireless network.](image-url)
also increased. Both the proposed schemes can capitalize even more on the use of denser topologies since the coding opportunities are increased. A subtle point is that the CLC-NC scheme introduces a message passing step that can increase the contention over the network by injecting non-data traffic like NC/802.11. However, this overhead is only experienced during the initial execution of the algorithm and not continuously for every group of coded packets.

B. Mobility

We considered a single mobile node for this experiment. The proposed algorithms exhibit considerably better performance as it can be seen in Fig. 3(b) and for different mobile speeds. A detailed look at the simulation traces indicates that when out-of-band acknowledgment reports are lost they severely affect performance since they correspond to groups of packets. This means that stale coded packets remain in the buffers without being acknowledged, something that prevents them of being used any further. With the analytically-driven algorithms, faster and more accurate feedback is provided to the network coding algorithm regarding the expected information rate before each broadcast transmission.

C. Algorithm Convergence

The throughput gain that is achieved in the complete network, versus the number of iterations of the distributed algorithm is presented in Fig. 4. The important message that we desire to convey with this figure is that with successive iterations we obtain a throughput gain that is increased monotonically. Furthermore, we see that the number of iterations needed for this set of experiments is independent from the network size. The dominant factor that determines the convergence speed is the node degree that expresses the number of neighbors of each node. This is something that is expected since the nodes have to wait for a longer period before they make a decision about the optimal PHY rate.

VII. CONCLUSIONS

In wireless ad hoc networks combining network coding with cross-layer adaptation is a necessity due to the exploitation of the broadcast nature of the channel. To attack this problem, we initially considered the above aspects in a joint analytical information rate model that quantifies the relative tradeoffs only for a single node. The developed model is used by an online opportunistic network coding algorithm that selects the optimal transmission parameters and group of packets that should be coded. Performance improvements were observed both for static and mobile scenarios.

REFERENCES